Journal of Engineering Physics and Thermophysics, Vol. 76, No. 4, 2003

MEASUREMENT OF THE RATE OF SEDIMENTATION OF FINE PARTICLES IN A PLATELIKE CENTRIFUGE

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UDC 532.584.22:53.082.13

The data of experimental investigation of the process of settling of fine particles in a platelike centrifuge have been given. The results obtained on sand and water have shown a nonmonotone character of the relative rate of sedimentation as a function of the total concentration of particles and the presence of the effect of a higher-than-average settling rate for the finest fractions. The methods of measurement of the sedimentation of fine particles are discussed, the experimental procedure is described, and systematic experimental errors are analyzed.

Sedimentation, or the settling of a solid phase, occurs in many processes involving a suspension. Thus, sedimentation results in the clarification of water, used in water purification and water preparation, which is largely reduced to the problem of separation of the solid suspended phase from the liquid phase. Sedimentation, or settling, favorably compares with other methods of purifying water, as, for example, filtration, floatation, etc., in its simplicity of implementation and low cost.

It is obvious that the rate of settling under gravity will be dissimilar depending on the mass of a particle. Having equated the force acting on a sphere in slow liquid flow to the difference between the mass force and the buoyancy force, we obtain a formula for determination of the rate of sedimentation of a single particle in the form [1, 2]

$$U_{0,i} = \sqrt{\frac{4d_i g}{3C}\frac{(\rho_{\text{p}} - \rho_{\text{liq}})}{\rho_{\text{liq}}}} \,. \tag{1}$$

In what follows, in analyzing the sedimentation of fine particles in a centrifuge, by the mass force we will mean the centrifugal force (instead of the gravitational force) and accordingly will consider the centrifugal acceleration equal to $\omega^2 r$.

For the coefficient of resistance C we can take the following expression [3]:

$$C = \frac{24}{\text{Re}_i} + \frac{4}{\sqrt{\text{Re}_i}} + 0.4 \,, \tag{2}$$

where the Reynolds number is defined as

$$\text{Re}_i = \frac{U_{0,i} d_i \,\rho_{\text{p}}}{\mu_{\text{liq}}} \,. \tag{3}$$

For low values of Re_i relation (2) can be bounded by the first term, and then (1) yields the Stokes formula for the rate of settling of a single particle of diameter d_i:

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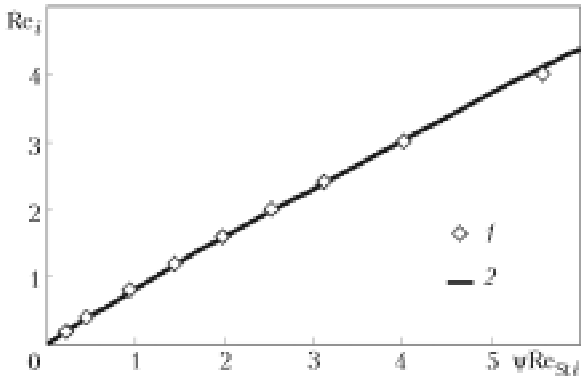


Fig. 1. Dependence of the rate of sedimentation of single particles, expressed by the relation of the parameters Re_i and $\mathrm{Re}_{\mathrm{St},i}$, in calculation: 1) from (5) and 2) from (6).

$$U_{\mathrm{St},i} = \frac{r d_i^2 \omega^2}{18\mu_{\mathrm{liq}}} (\rho_{\mathrm{p}} - \rho_{\mathrm{liq}}) . \tag{4}$$

The use of expression (4) is justified in most cases; for higher Re_i we can calculate the value (refined with account for (2) in relation to (4)) of

$$U_{0,i} = \psi \frac{r d_i^2 \omega^2 (\rho_{\mathrm{p}} - \rho_{\mathrm{liq}})}{18\mu_{\mathrm{liq}} (1 + 0.116 \sqrt{\mathrm{Re}_i} + 0.016 \, \mathrm{Re}_i)} = \psi \frac{U_{\mathrm{St},i}}{1 + 0.116 \sqrt{\mathrm{Re}_i} + 0.016 \, \mathrm{Re}_i} . \tag{5}$$

Another factor influencing the rate of sedimentation is the nonsphericity of the particles. One usually introduces for them the nonsphericity coefficient ψ, as has been done in (5), which acquires values of 0.5 (for long cylinders) and 0.8 (for cubes).

We can impart a more universal form to expression (5) by introducing the parameter $\mathrm{Re}_{\mathrm{St},i} = U_{\mathrm{St},i} d_i \rho_{\mathrm{p}} / \mu_{\mathrm{liq}}$. It is obvious from (5) that Re_i depends just on the combination $\psi \, \mathrm{Re}_{\mathrm{St},i}$ (Fig. 1). The relations

$$\mathrm{Re}_i = 0.83 \, (\psi \, \mathrm{Re}_{\mathrm{St},i})^{0.93} \quad \text{or} \quad \frac{U_{0,i}}{U_{\mathrm{St},i}} = 0.83 \psi^{0.93} \left(\frac{U_{\mathrm{St},i} d_i \, \rho_{\mathrm{p}}}{\mu_{\mathrm{liq}}} \right)^{-0.07} \tag{6}$$

hold with a high degree of accuracy. When $\mathrm{Re}_{\mathrm{St},i} = 4$, the correction to the value of the Stokes rate of sedimentation amounts to 25% for $\psi = 1$ now and to more than 50% for $\psi = 0.6$.

The actual rates of settling can significantly differ from the Stokes rate, especially for a thick polydisperse suspension. The reason is that the particles, moving with dissimilar velocities and being at small intervals from one another, can interact.

This work seeks to experimentally investigate the sedimentation of polydisperse suspensions and to establish the influence of particle interaction on this process.

Methods of Measurement of the Rate of Sedimentation. The simplest means of measuring the rate of sedimentation of fine particles is tracking the level separating a pure liquid and the sedimentation zone, i.e., the zone filled with the settling suspension, in a vertically arranged cylinder [4–9]. This method assumes the presence of a sharp boundary between the zones, which is not necessarily the case, especially for fine particles. Furthermore, the conclusions drawn on the basis of this method are based on the assumption that the suspension is homogeneous, i.e., there are no segregation processes. The latter is true for nothing but a monodisperse suspension. We also note that the sedimentation of particles about 10 μm in size, whose density differs only 2–3 times from that of the carrier liquid (water), occurs at rates of only several microns per second. To accelerate the process of sedimentation we can employ other mass forces, primarily, the centrifugal force, but an analysis of measurements in this case becomes more difficult than that in the case of settling in a vertical cylinder [9, 10].

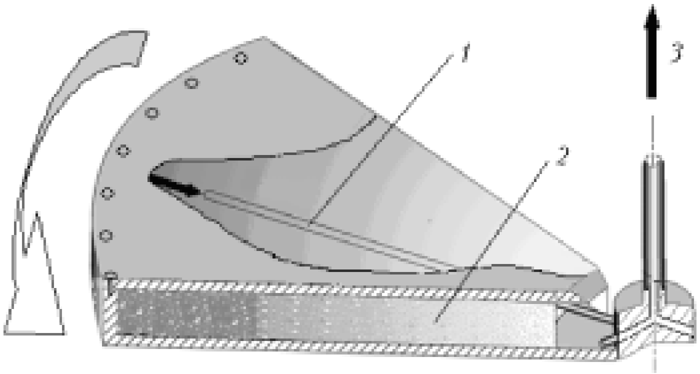


Fig. 2. Diagram of sampling of the suspension during the sedimentation in a rotating platelike centrifuge: 1) capillary; 2) suspension; 3) sampling.

The original (manometric) method of determination of the rate of settling indirectly by measuring the difference of the pressures at two points along the sedimentation direction with time has been employed in [11–13].

The difference of statistical pressure measured is in proportion to the average density of the suspension in the volume between the points. In a more or less stratified suspension, we observe deflection points on the curve of change of the pressure difference; these points correspond to the time of passage of the front of a certain size fraction through the measuring cell. In the case of a monodisperse suspension, these will be two times of passage of the sedimentation front through the first and second points where the pressure is measured. The distance between the points and the difference of the indicated times enable us to compute the rate of sedimentation.

Employing this method for measurement of the rate of sedimentation in a centrifuge, Bickert et al. obtained a number of interesting results. In particular, the existence of the effect of accelerated sedimentation in a certain range of concentrations of a suspension has been confirmed again. At the same time, it is difficult to use the manometric method for measuring the sedimentation rates of particles of different size, which form a polydisperse suspension, since the segregation of the suspension is indistinct as a rule. Furthermore, in processing the results, one has to numerically differentiate nonsmooth functions, which is one source of a systematic error.

A method of measuring the sedimentation rate that is based on granulometric analysis of samples at two points of the flow and is suitable for the case of a polydisperse suspension has been proposed in [14]. In their experiments, Kumar et al. employed a cylindrical column along which the suspension was moving.

Having written the mass balance for the liquid $U_0 S_0 [1 - c(0)] = U_z S_z [1 - c(z)]$ and a certain ith size fraction of particles in two cross sections of the flow $U_0 S_0 c_i(0) = U_z S_z c_i(z) + U_{s,i} c_i(z) S_z$, Kumar et al. arrive at the relation for the rate of sedimentation $U_{s,i}$ of the particles of the ith fraction:

$$U_{s,i} = \frac{U_0 S_0}{S_z} \left[\frac{c_i(0)}{c_i(z)} - \frac{1 - c(0)}{1 - c(z)} \right],$$

expressed by the velocity of motion of the liquid U_0 at the point $z = 0$, the areas S_0 and S_z of the cross sections, the concentrations of particles of the ith fraction c_i, and the total volume content of the particles c in these cross sections. By sampling in the initial cross section and in the cross section z and determining, at these points, the values of the total concentration of the solid phase and the concentrations of individual fractions, which are characterized by the size d_i of the particles contained in them, we can find the rate of sedimentation of the particles of the ith fraction.

In this method, the presence of sedimentation fronts is optional. Difficulties emerge here primarily because of the nonuniformity of the velocity field in the column. Nonetheless, Kumar et al. were able to measure, in such a way, the rate of sedimentation (or floatation) of particles in monodisperse and, which is particularly important, polydisperse suspensions. In particular, it has been inferred that fine fractions sediment much faster than is expected according to the Stokes law.

A similar method of measurement of the rate of sedimentation has been proposed in [15]. Here the suspension is supplied to a platelike centrifuge so fast that one can consider it to be uniformly distributed in the plate volume at the initial instant (Fig. 2).

One can sample the suspension through the capillaries connected to the tube on the centrifuge axis at different instants of time. The approximate solution of the problem for the time change in the concentration of particles, which is observed at a distance of r_p from the axis of rotation, yields a relationship between this concentration, the time, and the rate of sedimentation of the form

$$U_{s,i}\,(t, r_p) = -\frac{r_p}{2t} \ln \frac{c_i\,(r_p, t)}{c_{0,i}}\,. \tag{7}$$

Here, too, similarly to the method of Kumar et al. [14], information on the time change in the concentration of the particles of the fraction in question at the sampling point r_p is required to determine the sedimentation rate.

An advantage of the centrifugal method over the measurements in a column is the absence of suspension flow. In the entirely filled plate, we can have only a redistribution of the solid phase to the outer edge and the corresponding displacement of the liquid toward the centrifuge axis.

The experimental data obtained by these methods are discussed in [15, 16]. We only note that the results of the investigations carried out in [14] are in many respects confirmed in the case of employment of the centrifugal method.

Experimental Procedure. A theoretical foundation of determination of the sedimentation rate of a particle of diameter d_i in the centrifuge is the dynamic equation of concentration of the particles of a certain ith fraction

$$\frac{\partial c_i}{\partial t} + \frac{1}{r} \frac{\partial\,(r c_i\, U_i)}{\partial r} = 0 \tag{8}$$

with the corresponding initial condition and boundary condition. This equation has been considered in the field theory of sedimentation since the 1950s [4]. If the rates of sedimentation depend on the concentrations of all fractions, the theory of sedimentation is described by a system of, generally speaking, nonlinear undecomposed equations. The characteristics of Eq. (8) form the system of ordinary differential equations

$$dt = \frac{dr}{r\Phi'\,(c)} = -\frac{dc_i}{2\Phi\,(c_1, c_2, ..., c_i, ...)}\,, \tag{9}$$

where

$$\Phi = c_i \frac{U_{s,i}}{r}\,, \quad \Phi' = \frac{\partial \Phi}{\partial c_i}\,. \tag{10}$$

For a rarefied monodisperse suspension, we have $\Phi = U_{St,i} c_i / r$ and solutions of the equation of characteristics (9) yield the boundary of the sedimentation front $r_{fr} = r_0 \exp\,(U_{St,i} t / r_p)$ and the value of the concentration of particles in the sedimentation zone ($r_p > r_{fr}$) in the form

$$c_i = c_0 \exp\,(-\,2U_{St,i} t / r_p)\,. \tag{11}$$

When the dependence of the rate of sedimentation on the concentration of all the fractions is unknown, Eqs. (9) and (10) yield

$$\Phi = -\frac{1}{2} \frac{dc_i}{dt} = c_i \frac{U_{s,i}}{r}\,. \tag{12}$$

Equation (12) gives the expression for the rate of sedimentation of particles of the ith fraction, which is written in terms of the concentration change of this fraction at the selected point r_p:

$$U_{s,i} = -\frac{r_p}{2} \frac{d \ln c_i}{dt}\,. \tag{13}$$

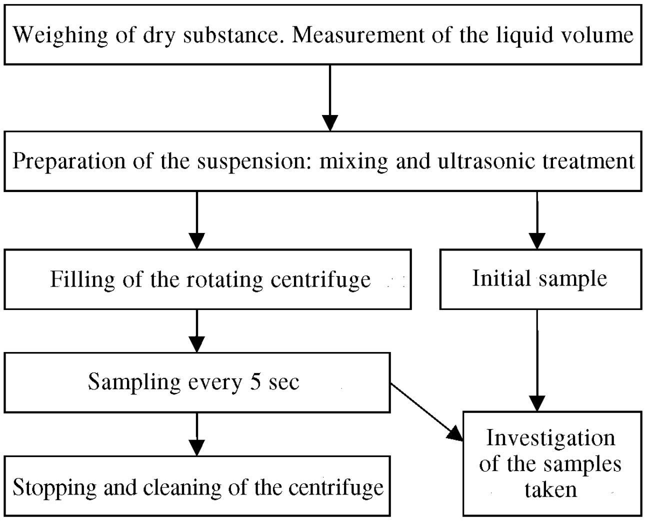


Fig. 3. Course of the experiment.

Approximation of (13) with the use of the initial values results in expression (7), which was employed in [15]. Strictly speaking, it holds only for a monodisperse suspension. It is more reasonable to employ the differential form of (13).

In the experiments conducted in the present work, we use a procedure which is similar to that described in [14, 15] and is based on sampling the sedimenting suspension at a certain site and at certain instants of time. To find the rates of sedimentation of particles in a polydisperse suspension it is sufficient to investigate the samples taken for the size distribution of the particles and the total concentration of the solid phase.

The very slow motion of micrometer particles makes the experiment longer and contributes to an error growth in processing the data of measurements. This problem is solved in [14] by pumping the suspension through a precipitation column; in this case the particles are pushed by the motive force. However, this method brings about a considerable error in measurements of the sedimentation rate due to the inequality of the velocities of the moving suspension at the walls and at the center of the column. Such errors are excluded in the procedures of [13, 15], where a centrifuge is employed. Here particles are moving horizontally under the action of the centrifugal force, exceeding the gravitational force tens of times, which allows experiments with particles smaller than 1 μm. The medium remains stationary.

In the centrifuge used (Fig. 2), the vessel with capillaries is spinned in such a manner that the centrifugal force exceeds the gravitational force nearly 50 times (for a rotational velocity of 750 rpm and a radius of the centrifuge of 0.08 m).

The course of the experiment is presented in Fig. 3. Each experiment begins with preparation of a suspension, which involves measuring accurate volumes of a liquid (tap water, pH 7.2) and a solid substance (sand, content of silicon 98%, density $\rho_p = 2600$ kg/m^3) with the aim of establishing the desired concentration of the suspension. Next is a thorough dispersion of the suspension. We added surfactants to the suspension with the aim of excluding the agglomeration of particles, which improves the accuracy and reproducibility of the experiments.

The dispersion of the solid fraction and the surfactants is carried out for 10 min using an impeller and an ultrasonic bath. Once the process of preparation of the suspension is completed, we take a sample in which we determine the initial size-distribution function of the dispersed particles.

Next, the suspension is poured as quickly as possible into the already spinned centrifuge, and the counting of time and sampling begin as soon as the entire volume is filled. We take 5 to 10 samples with a volume of 2 ml each in the course of the experiment. Such an amount is sufficient to investigate the suspension for the total concentration of the solid phase and the particle-size distribution.

The total concentration is determined by weighing a certain volume of the liquid according to the formula

$$c = \frac{m_{\text{sus}}/V_{\text{sus}} - \rho_{\text{liq}}}{\rho_p - \rho_{\text{liq}}} .$$

The volume of the suspension is measured accurate to ±0.0005 ml using a pycnometer, and weighing is carried out accurate to ±0.000005 g on an analytical balance.

To obtain the particle-size distribution we employ a Mastersizer 2000 device (Malvern Corp.), whose principle of operation is based on measuring the intensity distribution of a scattered laser beam transmitted by the suspension. The value of the concentration of each fraction of the solid substance is restored from solution of the inverse problem of scattering theory.

Using the data obtained, we can observe the manner in which the concentration of the *i*th fraction changes in the course of the experiment. From the formula (13) written in the form

$$U_{s,i} = -\frac{r_p}{2\,(t_k - t_{k-1})} \ln \frac{c_i\,(t_k)}{c_i\,(t_{k-1})}\,,$$

we compute the rate of sedimentation of each fraction at the instant of time $(t_k + t_{k-1})/2$.

The obtained values of the particle velocities are compared to the velocities calculated in accordance with the Stokes theory.

Analysis of the Possible (Systematic) Experimental Errors. Apart from the inevitable statistical errors that can be decreased by repetition of measurements (they are within 5% in our experiments), of great importance are systematic errors that can be identified only by analyzing the process of measurement. Below we give some evaluations of inaccuracies caused by the duration of sampling, the inertia of particles, the distortion of the circular symmetry of concentration fields, etc.

Evaluation of the Time of Filling of a Free Centrifugal Space. It takes some time to pour a suspension into a centrifuge, during which the suspension flows out through a funnel and fills the free space of the apparatus. The time of flowing out through the funnel is about $t_{\text{fun}} = \sqrt{\frac{2h_0}{g}} \approx \frac{V_{\text{sus}}^{1/2}}{g^{1/2}\tan\alpha^{1/6}}$. Taking $V_{\text{sus}} = 150 \cdot 10^{-6}\ \text{m}^3$, $g = 10\ \text{m} \cdot \text{sec}^{-1}$, and $\alpha = \pi/4$, we obtain the estimate $t_{\text{fun}} \approx 0.07$ sec.

We can evaluate the filling time, formulating the problem of motion of the liquid front under the action of the centrifugal force in the form

$$\frac{d^2 r}{dt^2} = \omega^2 r\,, \quad r\big|_{t=0} = r_0\,, \quad \left.\frac{dr}{dt}\right|_{t=0} = 0\,,$$

whence the filling time will be found as $t = \omega^{-1}$ arch (R/r_0). For $R = 0.08$ m, $r_0 = 0.01$ m, and $\omega = 78.5\ \text{sec}^{-1}$ the filling time is $t = 0.026$ sec.

The estimates obtained point to the fact that, in filling the centrifuge, the longest is the period of transfer from the glass to the funnel and the total time of filling is, probably, no longer than 0.1 sec, which is much lower than the time interval to the instant of the first sampling, equal to 5 sec.

Error in Determination of the Concentration in the Sampling. The suspension is sampled during a certain time Δt. Since the concentration of the suspension in the centrifuge changes with time, the concentrations will be different at the beginning of the sampling and at the end of it; as a result, we will observe a certain average concentration. The resulting error can be evaluated using the solution of Eq. (11) for the concentration change $c_i(t) = c_{i0} \exp\left(-\frac{2U_{\text{St}}}{r_p}t\right)$. From the approximation $c(t + \Delta t) = c(t) + \frac{\partial c}{\partial t}\Delta t + \ldots$, we obtain the average value of the concentration $\bar{c}_i(t) = \bar{c}_i(t)\left(1 - \frac{U_{\text{St}}}{r_p}\Delta t\right)$. Then the relative error can be evaluated as

$$\varepsilon = \left|\frac{\bar{c}_i\,(t) - c_i\,(t)}{c_i\,(t)}\right| = \frac{U_{\text{St}}}{r_p}\Delta t = \frac{d^2\omega^2}{18}\,\frac{(\rho_p - \rho_{\text{liq}})}{\mu_{\text{liq}}}\,\Delta t\,.$$

TABLE 1. Relative Errors in Determination of the Concentration of Particles of Different Fractions in Sampling

d, μm	1	5	10	15	20
ε	0.0005	0.013	0.05	0.12	0.2

As is seen from the formula obtained, the determination error for the concentration at the instant of time t is in proportion to the square of the particle diameter, the square of the angular rotational velocity of the centrifuge, the density difference, and the sampling time, and it is in inverse proportion to the suspension viscosity. For $\rho_p = 2500$ kg/m^3, $\rho_{liq} = 1000$ kg/m^3, $\mu_{liq} = 10^{-3}$ Pa·sec, $\omega = 78.5$ sec^{-1}, and $\Delta t = 1$ sec, we obtain the estimates given in Table 1 for particles of different size.

Thus, for fairly large particles the concentration measured can be much lower than the true concentration.

Error in Determination of the Concentration because of the Broken Circular Symmetry in the Vicinity of a Capillary. The suspension is sampled using a capillary the radius of whose cross section is equal to r. In sampling, a suspension of volume V_1 is drawn into the capillary from the vicinity of its inlet. This vicinity can be approximated by a sphere of radius $R_1 = \sqrt[3]{\frac{3}{4\pi} V_1}$. Since the settling particles move along the capillary surface, in the wake of the capillary there forms a region of pure liquid whose volume $\pi r^2 R_1$ must be negligible as compared to the volume of the sample taken, $V_1 = \frac{4}{3}\pi R_1^3$. Then the expression for the error caused by the sampling of the volume not containing the true number of particles will be $\varepsilon = \frac{3}{4}\left(\frac{r}{R_1}\right) \approx 2\frac{r^2}{V_1^{2/3}}$. This yields the requirement imposed on the radius of the capillary cross section $r \ll R_1 \approx (V_1)^{1/3}$. For $V_1 = 4 \cdot 10^{-7}$ m^3 and $r = 3.5 \cdot 10^{-4}$ m we have $\varepsilon = 0.004$.

Error Caused by the Inertia of Particles. In sampling, the sedimenting particles are entrained by the liquid drawn into the capillary and they change their trajectory. Larger particles as more inert ones can have no time to be entrained into the capillary. As a consequence, their relative number can be understated in the sample taken. It is obvious that the characteristic time of relaxation of a particle to the changing flow of the liquid in the experiment must be shorter than the sampling time: $t_r = \frac{1}{18}\frac{\rho_p}{\mu_{liq}} d^2 \ll \Delta t$. We can improve the estimate on the basis of the equation of motion of a particle finding itself in the flow of the liquid drawn into the capillary and settling under the action of centrifugal forces. Let U be the velocity of the liquid, which points away from the rate of settling of the particle; then the velocity of motion of the particle can be computed from the equation $\frac{dU_s}{dt} = t_t^{-1}\,(U - U_s - U_{St})$ with the initial condition $t = 0$ and $U_s = -U_{St}$. The solution has the form $U_s = U\,(1 - \exp(-t/t_r)) - U_{St}$. The particle can be drawn into the capillary if U_s becomes larger than zero, i.e., the following relations must hold for the particle velocity:

$$t_r \ln\left(\frac{U}{U - U_{St}}\right) \ll \Delta t \quad \text{and} \quad U - U_{St} > 0\,.$$

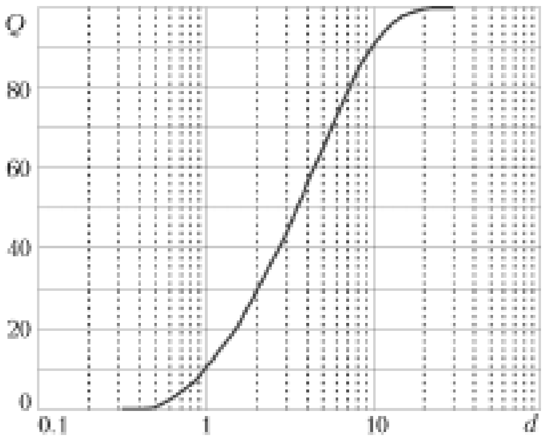
The latter inequality determines the possibility of drawing the liquid into the capillary. The average velocity of a suspension drawn in through the capillary can be evaluated as $V_1/\pi r^2 \Delta t$. Then we obtain the estimate for the minimum permissible volume of the suspension drawn in through the capillary:

$$V_1 > V_{sam} = \frac{d^2\omega^2 r_p}{18}\frac{(\rho_p - \rho_{liq})}{\mu_{liq}}\pi r^2 \Delta t\,.$$

A number of values for the error are given in Table 2 for $\rho_p = 2500$ kg/m^3, $\rho_{liq} = 1000$ kg/m^3, $\mu_{liq} = 10^{-3}$ Pa·sec, $\omega = 78.5$ sec^{-1}, and $r_p = 0.08$ m. All these values are much lower than the real volume of the sample drawn in through the capillary ($4 \cdot 10^{-7}$ m^3).

TABLE 2. Minimum Permissible Volumes of the Suspension Drawn-in through the Capillary as Functions of the Particle Size

d, μm	1	5	10	15	20
V_{sam}, m^3	$1.4 \cdot 10^{-11}$	$3.5 \cdot 10^{-11}$	$1.4 \cdot 10^{-9}$	$3.15 \cdot 10^{-9}$	$5.5 \cdot 10^{-9}$



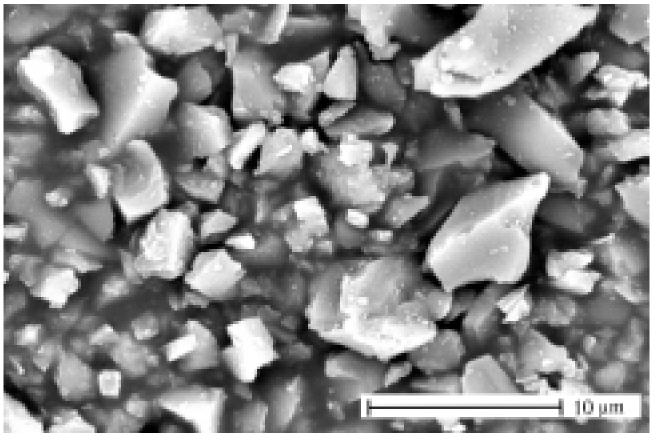


Fig. 4. Total volume-distribution function of the particles of the material under study (Sikron SF-500) vs. particle diameters. Q, %; d, μm.

Fig. 5. Photograph of the grains of the material under study, made with an electronic microscope.

Disturbance of the Position of Concentration Isolines Due to the Sampling. In taking the sample of a suspension at the sampling point r_{p}, the volume of the contents of the centrifuge decreases by $6V_1$. The sedimentation front is displaced to the periphery by a distance which can be found from the relation for the conservation of volumes $6V_1 = \pi h(r_{\text{d}}^2 - r_{\text{p}}^2) \approx 2\pi h r_{\text{p}}(r_{\text{d}} - r_{\text{p}})$, where r_{d} is the displaced position of the concentration isoline after the sampling, which was at a distance r_{p} before the sampling. Then the relative drop of the concentration in the vicinity of the capillary mouth (error) will be

$$\varepsilon = \frac{\Delta c}{c} = \frac{r_{\text{d}} - r_{\text{p}}}{r_{\text{p}}} \approx \frac{6V_1}{2\pi h r_{\text{p}}^2} .$$

For a centrifuge height of $h = 0.01$ m, $6V_1 = 2.4 \cdot 10^{-6}$ m^3, and $r_{\text{p}} = 0.07$ m, we have $\varepsilon = 0.08$. After the five samples taken, the total error in determining the concentration of particles will amount to 4%.

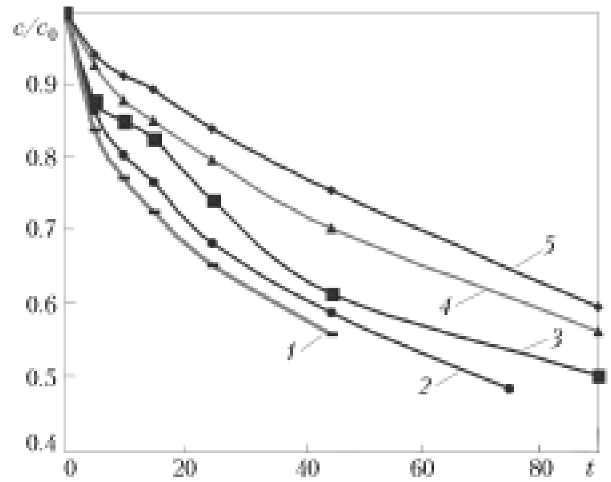
Thus, the level of systematic error in measurements of the concentration of particles is mainly determined by the rate of sampling. The decrease in the sampling time due to the increase in the capillary radius, unfortunately, has its bounds, since a large error is introduced into the value of the volume of the suspension taken in the case of wide capillaries.

Measurement Results. To study the interference of particles in settling we have conducted experiments for different initial concentrations of the solid fraction. The dispersed phase in all the conducted experiments was ground sand (average size of the particles of the material under study $d = 3.6$ μm) whose total volume-distribution function $Q(d)$ is presented in Fig. 4. As is seen in Fig. 5, the particle shape is far from spherical. In evaluations of the rate of settling of a single particle from formula (6), the nonsphericity coefficient ψ was taken to be 0.6.

The lowest measured concentration is equal to one volume percent, which is determined by the sensitivity of the device for measuring the particle-size distribution. The values of the concentrations are bounded above only by the condition of fluidity of the suspension (about 20 vol.%).

The samples taken in the experiments conducted have been analyzed to give two families of dependences.

The first family represents a change in the total concentration of the solid phase at the sampling point $r_{\text{p}} = 0.07$ m in the course of the experiment (Fig. 6). From these curves we can infer that the higher the concentration of



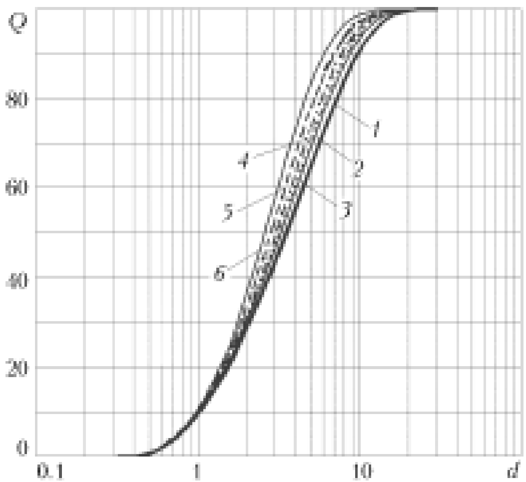
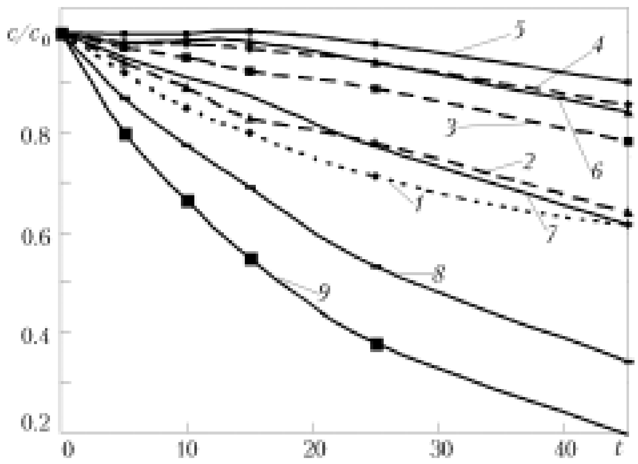


Fig. 6. Change in the relative total concentration of the solid phase with time for suspensions with different initial volume contents of a solid phase: 1) 1; 2) 5; 3) 10; 4) 15, and 5) 20%.

Fig. 7. Change in the size-distribution function of particles with time (concentration 10 vol.%): 1) $t = 0$, 2) 5; 3) 10; 4) 15; 5) 25, and 6) 45 sec.



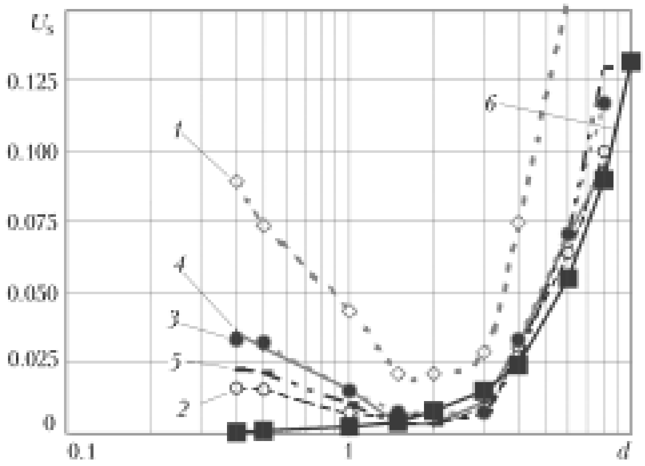


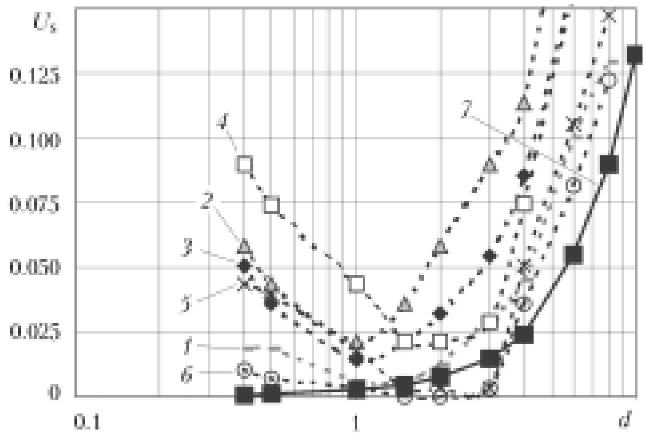
Fig. 8. Change in the concentrations of individual fractions in the course of the experiment: 1) $d = 0.4$; 2) 0.5; 3) 1; 4) 1.5; 5) 2; 6) 3; 7) 4; 8) 6, and 9) 8 μm. t, sec.

Fig. 9. Rates of sedimentation for different sampling times (initial concentration of particles 10 vol.%): 1) 2.5; 2) 7.5; 3) 12.5; 4) 20; 5) 35 sec; 6) single particle, according to Eq. (6). U_s, cm/sec; d, μm.

the suspension, the more slowly it sediments. In the literature, this is traditionally related to the increase in the effective viscosity and the density of the suspension and to the generation of the counterflows of a liquid displaced by settling particles [5, 6, 17]. These mechanisms are easily interpreted, on the basis of the Stokes formula (4), by substitution of the effective values of the suspension parameters. The formula thus derived in [17] has the form

$$\frac{U_{h,i}}{U_{St,i}} = (1-c)^2\left(1-\frac{c}{0.6}\right)^{3/2} . \qquad (14)$$

The second family of curves is a change in the particle-size distribution function with time (Fig. 7). As an example, we give here distribution curves corresponding to an initial concentration of the solid phase of 10%. These



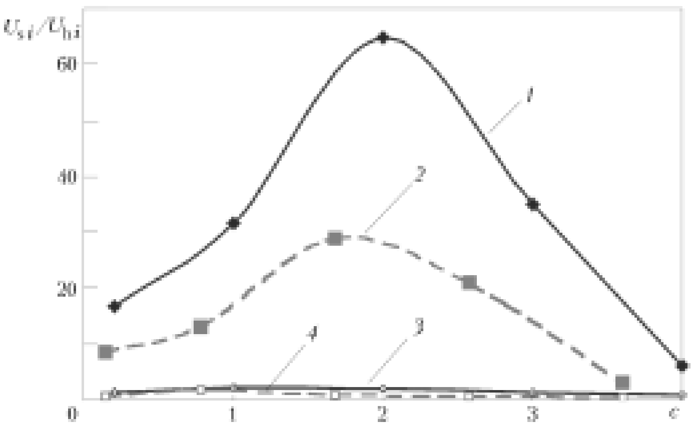


Fig. 10. Rates of sedimentation for different initial concentrations (sampling time 5 sec): 1) 1; 2) 1.5; 3) 5; 4) 10; 5) 15; 6) 20 vol.%; 7) single particle, according to Eq. (6). U_s, cm/sec; d, μm.

Fig. 11. Influence of the total concentration in the relative rate of settling of fine fractions: 1) d = 0.5 and t = 2.5, 2) 0.5 and 12.5, 3) 4 and 2.5, and 4) 4 μm and 12.5 sec.

dependences can be transformed to a concentration change of each fraction of the solid substance according to the formula

$$c_i = c\Delta m_i \, , \tag{15}$$

where $\Delta m_i = [Q(d_{i+1}) - Q(d_i)]/100$.

The time changes in the concentrations of individual fractions are given in Fig. 8, where the particle size of the corresponding fraction of a solid substance is indicated. To make the representation more lucid we give in the figure only some of the fractions measured.

Knowing the time dependence of the change in the concentration of individual fractions, we can calculate sedimentation rates for particles of different size using (13). Figure 9 shows the curves corresponding to different instants of sampling time and the curve of rates calculated from the Stokes formula. The given results yield that fine fractions of particles ($d_p \le 2$ μm) have sedimentation rates higher than particles of medium-size fractions (2 μm $\le d_p \le$ 4 μm) at all instants of time. The dependence of the rate of sedimentation of the particle size is nonmonotone in character at each instant of time. For particles with a size of about 1–2 μm the rates of sedimentation are minimum. As far as the time behavior is concerned, the $U_s(d)$ curves approach the Stokes curve, which is explained by the decrease in the total concentration of the suspension (Fig. 5), because of which the interaction of particles decreases.

Figure 10 shows the curves of experimental rates of sedimentation for different initial concentrations of the solid fraction at a fixed instant of sampling time t = 5 sec. From these dependences it is clear that the differences between the Stokes rates and the measured rates attain maximum values for the experiment conducted with the suspension with an initial concentration of 10 vol.%. This is illustrated once again in Fig. 11, where the ratio of the measured rate of sedimentation to the Stokes rate is shown as a function of the values of the total concentration of the suspension at the sampling point. For particles with a selected size of 0.5 and 4 μm the dependence has a typical form with its maximum at a certain value of the total concentration. For larger particles it lies at lower values of the concentrations than for finer particles.

To explain the dependences (see Fig. 11) we use the effect (described in [16–19]) of acceleration of fine particles by large ones in addition to the mechanism of deceleration of sedimentation because of the growth in the effective viscosity and density of the suspension. The indicated effect implies that fine particles, arriving at the boundary layer of large particles, are partially entrained by the flows generated by the large particles, thus accelerating their motion in the direction of sedimentation. For a bidisperse suspension we are able [16] to obtain a quantitative estimate of

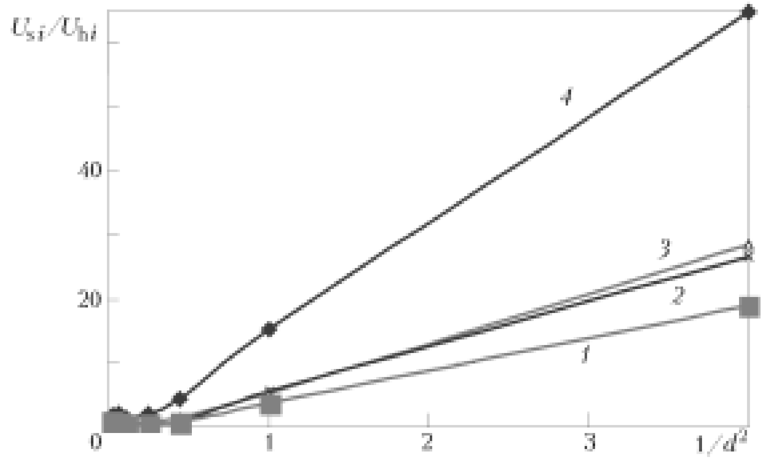


Fig. 12. Relative rate of settling vs. diameter of particles with an initial concentration of: 1) 7; 2) 7.7; 3) 8.4; 4) 10 vol.%.

the growth in the sedimentation rate of fine particles (they are denoted below by the subscript f) because of the presence of large particles (subscript g) of the form

$$U_{\mathrm{s,f}} = U_{\mathrm{h,f}}\left[1 + 1.35c_{\mathrm{g}}^{1/3}\left(\frac{d_{\mathrm{g}}}{d_{\mathrm{f}}}\right)^{2}\right]. \tag{16}$$

For qualitative considerations we can also use this estimate in the case of a polydisperse suspension, meaning a certain diameter of particles from the arbitrarily singled-out large spectrum by d_{g} and accordingly their concentration (we will take it to be c as a first approximation) by c_{g}.

Combining formulas (14) and (16), we obtain

$$U_{\mathrm{s,f}} = U_{\mathrm{St,f}}\left[1 + 1.35c^{1/3}\left(\frac{d_{\mathrm{g}}}{d_{\mathrm{f}}}\right)^{2}\right](1 - c)^{2}\left(1 - \frac{c}{0.6}\right)^{3/2}. \tag{17}$$

According to formula (17), $U_{\mathrm{s,f}}$ as a function of c passes through the maximum nearly at $c \approx 8$–10%. For low c the acceleration effect is insignificant, since the number of large particles is inadequate to accelerate a fine fraction; for higher concentrations the deceleration (14) is more pronounced.

The height of the maximum depends on the relation $d_{\mathrm{g}}/d_{\mathrm{f}}$. Measurements performed at more recent instants of time are characterized by the fact that large fractions have already sedimented and, consequently, in this case $d_{\mathrm{g}}/d_{\mathrm{f}}$ is smaller than in the earlier measurements. In accordance with this, we have strongly different curves for d = 0.5 and 4 μm. Finally, degeneration of the maximum of the curves (see Fig. 11) in the course of the experiment is explained by the fact that the total concentration of the suspension drops with time and the interaction of particles disappears.

Figure 12 illustrates the feasibility of the dependence (16) of the sedimentation of fine particles on their diameter. Here the curves of $U_{\mathrm{s},i}/U_{\mathrm{h},i}$ as a function of $1/d^2$ are plotted for the instant of time t = 5 sec; they correspond to different initial concentrations of the suspension. In the region of small d, the curves become straight lines whose slope coefficient, in accordance with (16), is the smaller, the lower the concentration of large particles.

CONCLUSIONS

1. We have developed a method making it possible to determine the rates of sedimentation of particles of individual size fractions on the basis of measuring the dynamics of the total concentration of the suspension and the concentration of individual fractions at a certain point of the centrifuge.

2. The results obtained show a strong influence of particles of different size on the rate of their sedimentation. In particular, it has been shown that fine particles are accelerated by large particles. This effect is the most pronounced for a 10% total volume concentration of the suspension.

3. A theoretical result on the increase in the rate of sedimentation of fine particles in a polydisperse suspension in inverse proportion to the square of the size of these particles has been confirmed experimentally.

4. The mechanisms of deceleration of sedimentation which are significant for thick suspensions have been revealed and discussed.

NOTATION

c, volume concentration of particles; C, coefficient of resistance; d, particle diameter, m; g, acceleration of gravity, m/sec^2; h, height of the centrifuge, m; h_0, height of the level of the suspension in a funnel, m; m, mass, kg; Δm, mass fraction; r, distance from the centrifuge axis, m; R, radius of the centrifuge, m; R_1, radius of an arbitrary sphere for the sample volume taken, m; S, cross-sectional area of the column, m^2; t, time, sec; $Q(d)$, size-distribution function of particles, %; U, velocity of motion of the suspension, m/sec; U_{s}, rate of sedimentation, m/sec; $U_{\mathrm{St},i}$, rate of settling of a single particle according to Stokes, m/sec; $U_{0,i}$, rate of settling of a single particle, m/sec; $U_{\mathrm{s},i}$, rate of sedimentation of the ith fraction of particles in the suspension, m/sec; V, volume, m^3; V_1, volume of the sample taken using one capillary, m^3; z, coordinate, m; α, angle of opening of the funnel; μ, dynamic viscosity, Pa·sec; ρ, density, kg/m^3; ω, angular rotational velocity of the centrifuge, sec^{-1}; ε, error; ψ, nonsphericity coefficient. Subscripts: 0, initial value, initial cross section; d, displacement; f, fine; fr, sedimentation front; fun, funnel; g, large; h, constrained (hindered) settling; i, fraction number; k, number of the sample taken; liq, liquid; p, particle; r, relaxation; s, settling; sus, suspension; z, running cross section; sam, sample; St, Stokes.

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